A Simple ODE Model for ICF Gain by Volume Ignition

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first light

Introduction

Simplified models exist for hot spot ignition, where a burn wave propagates into surrounding cold fuel, and for equilibrium ignition, where a high-Z tamp traps radiation and provides confinement, although the latter are less fully developed and significantly more complex. A simplified model for volume ignition is not known to the authors. These simple models cannot be relied on for accurate target design but they can inform the design process at a fundamental level.

The present work develops a simple ODE model for volume ignition, verifying against prior holistic simulation studies with good results.

One aspect necessitated the use of an ODE model, and this is modelling the burn fraction. At lower temperatures the power balance may be positive but the heating rate may be slow, such that the confinement time becomes the limiting factor. The simple burn fraction formula cannot be applied and an ODE approach was needed to capture this trade-off.

An additional complication is degeneracy, which is important to understanding the maximum attainable gain. At some point disproportionately more energy is required to reach higher density. Further complication arises from modelling the trapping of radiation, which breaks the normal areal density scaling. This means the parameter space to be explored is 3D rather than 2D.

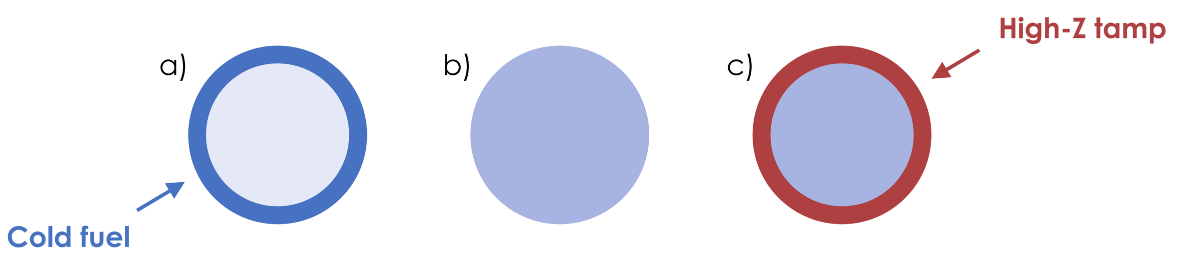

Fig. 1: A sketch illustrating three basic fuel configurations for ICF:
a) isobaric hot spot,
b) volume ignition,
c) equilibrium ignition.

Model

The thought experiment under consideration is a sphere of equimolar DT at a uniform temperature and density and surrounded by vacuum. **On release from this initial condition a rarefaction immediately begins to propagate inwards from the outer surface.** This quenches both the temperature and density; the rarefied material can be assumed to effectively disappear, making no further contribution to the reaction. **Meanwhile, the central region of the plasma is being heated (or cooled) through a balance of alpha heating, radiation loss and conduction loss, and the DT is being consumed.**

The model therefore has three dynamic parameters, the temperature, the partial density of DT and the radius of the fusing plasma. The rate equations are,

$$\frac{dT}{dt} = \frac{f_\alpha W_\alpha - W_C - W_R}{(3/2)\Gamma\rho}$$

$$\frac{d\rho_{DT}}{dt} = -\frac{\rho_{DT}^2\langle\sigma v\rangle}{2m_{DT}}$$

$$\frac{dr}{dt} = -\sqrt{c_{ideal}^2 + c_{fermi}^2}$$

where W_α and W_C are given by Atzeni eqs 4.2 and 4.10 respectively. The total density is assumed to be constant. The reaction term itself is calculated with the partial density, but all other terms use the total density. This is akin to modelling the mixture of D, T and He as equimolar DT from the point of view of the plasma properties.

The radiation term, W_R, is given by,

$$W_R = \sigma T^4\left(1 - e^{-r_{dirac}/l_p}\right)$$

where r_{dirac} is the Dirac chord and l_p is the Planck mean free path given by Atzeni eq 10.117.

The yield is tracked with an additional, uncoupled rate equation,

$$\frac{dY}{dt} = 5W_\alpha V$$

where the factor of five recognises that the alphas are only one fifth of the total energy.

The internal energy of the plasma is needed to calculate fuel gain and is found by adding the ideal gas and fermi gas expression in quadrature.

$$e = \sqrt{e_{ideal}^2 + e_{fermi}^2}$$

This expression accounts for the investment of energy in degeneracy, which limits the attainable gain. Not that the rate equation for temperature uses only the ideal gas expression however. Using the quadrature expression the energy equation cannot be rearranged to give temperature.

Validation

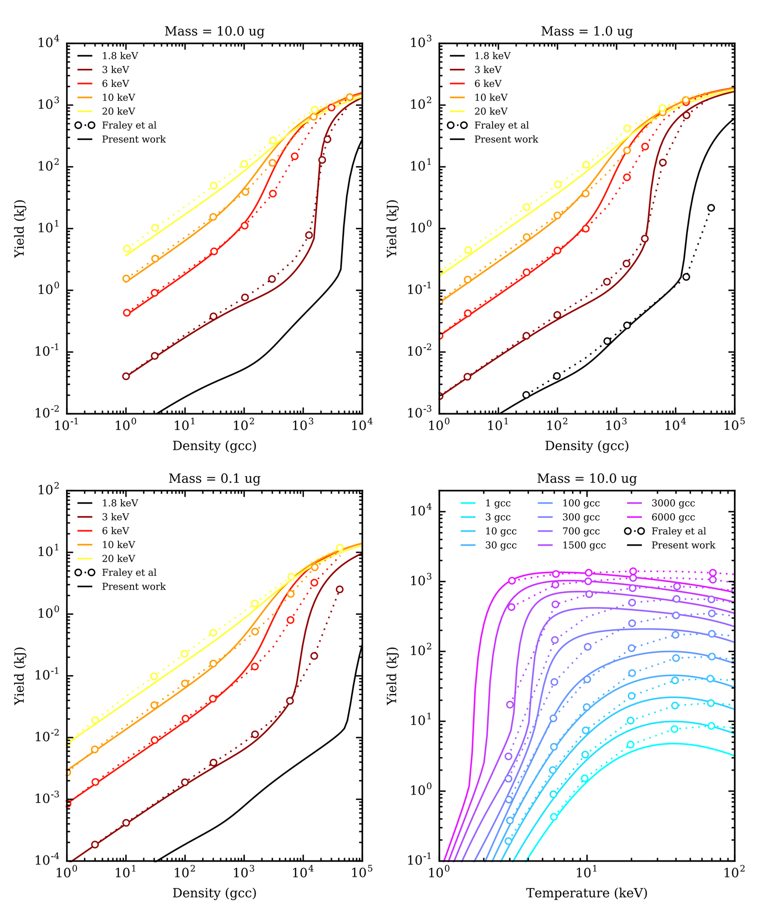

Fig. 2: Replication of yield curves from Fraley et al, Phys Fluids, 1974, fig 7.

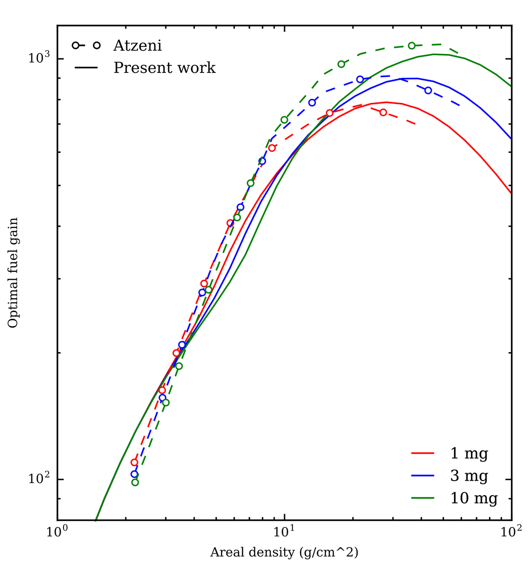

Fig. 3: Replication of the optimum gain curves given by Atzeni, Jpn. J. App. Phys, 1995

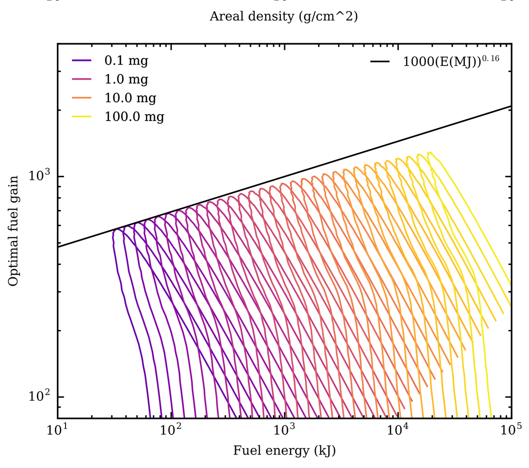

Fig. 4: Replication of the limiting gain curve, showing the maximum of the optimum gain over all fuel masses, given by Atzeni, The Physics of Inertial Fusion, 2004

Results

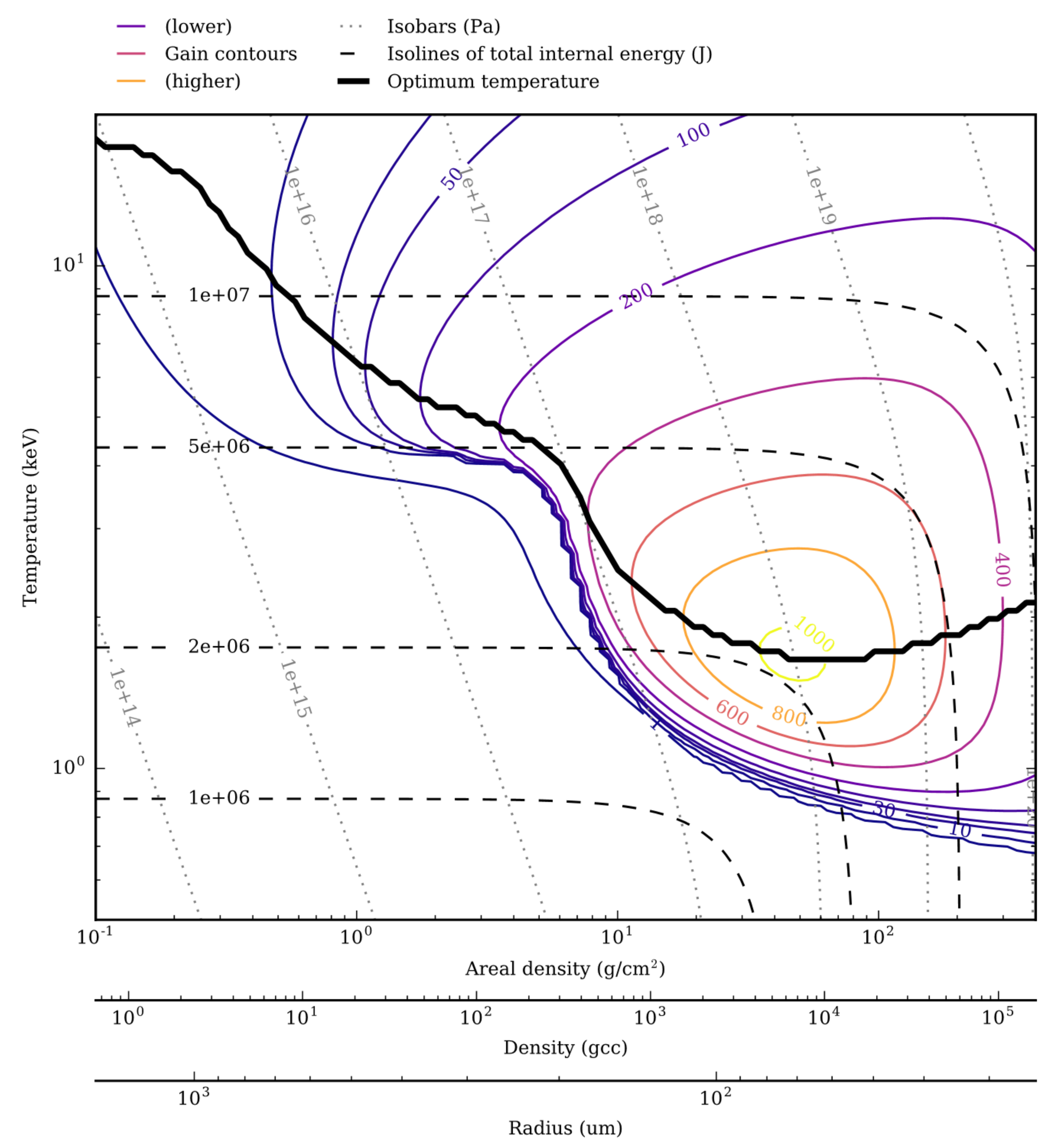

Fig. 5: An extended replication of Atzeni fig 5.11. Fuel mass is 10 mg.

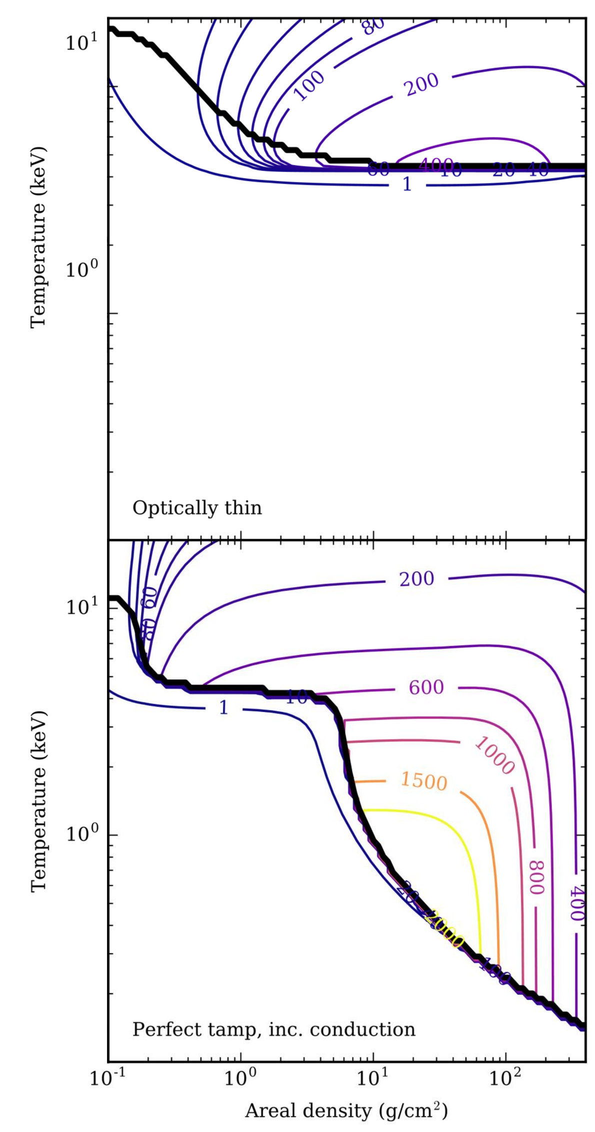

Fig. 6: top: effect of turning off radiation trapping, bottom: effect of infinite confinement time

Fig 5 shows contours of gain in temperature – density space. For each density there is on optimum temperature where gain is maximised. Above this temperature the additional energy investment does not deliver significant enough improvement in burn to increase gain.

At high density the pressure contours can be seen to deviate downwards, this is due to degeneracy. The gain is correspondingly reduced by the energy investment in said degeneracy.

On the gain boundary there are three regimes:

1. **High T, low density – conduction loss dominates, and even when igniting confinement time limits gain**
2. **Mid T, mid density – radiation is beginning to be trapped whereas the $T^{5/2}$ scaling of conduction has reduced it's importance**
3. **Low T, high density – confinement time dominates, but if ignition takes place high gain is achieved**

Fig 6 shows the effect of switching to an optically thin model, illustrating the second regime of behaviour, and the effect of a perfect tamp where the radius is considered fixed, illustrating the third regime. With this fictional perfect tamp, ignition could in fact take place from an initial condition substantially below 1 keV.

Finally, if we examine a given gain contour it can be seen that the point where pressure is minimised is not the same as that where total internal energy is minimised.

Fig 7 shows contours of optimum gain for many fuel masses. It shows that maximum gain increases as fuel mass increases, and that the required temperature is the same but the required density is less.

The plot also allows holistic assessment of both matters of target dynamics and reactor engineering. Three target constraints:

1. convergence ratio less than 30,
2. final size larger than 10 µm,
3. target gain of more than 10,

and three reactor constraints:

1. driver less than 100 MJ,
2. energy released less than 10 GJ,
3. repetition rate no faster than 1 Hz for 100 MW output,

have been overlaid. The region where no constraint applies is the "island of viability". For volume ignition it is clear that extremely high density is required. Additionally, **if coupling efficiency is less than 2%, there is no island of viability at all.**

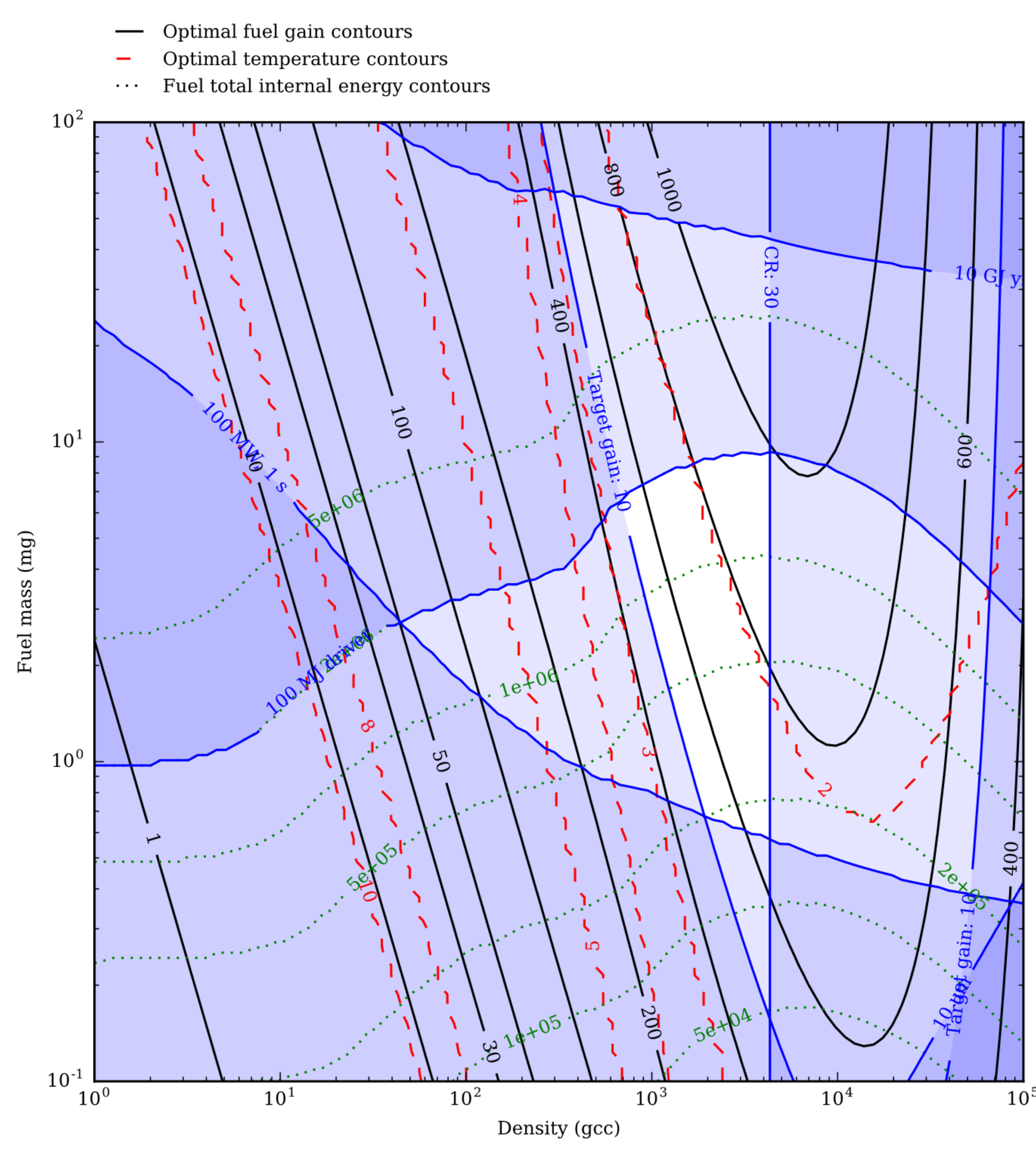

Fig. 7: A contour plot of optimised temperature and gain. The areas shaded blue are disallowed for practical power production, leaving only the "island of viability" in the centre. Coupling eff is 2%.

Conclusion

A simple ODE model has been developed for volume ignition and verified against detailed simulation work of previous authors. The results allow rapid exploration of the design space for power production. Volume ignition is shown to require very high density, but additionally to also require coupling efficiency of greater than 2% for any practical power production system to exist.

Next steps will examine similar models for hot spot and equilibrium ignition and compare.

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