Benchmarking Hytrac simulations of the Richtmyer-Meshkov Instability at First Light Fusion

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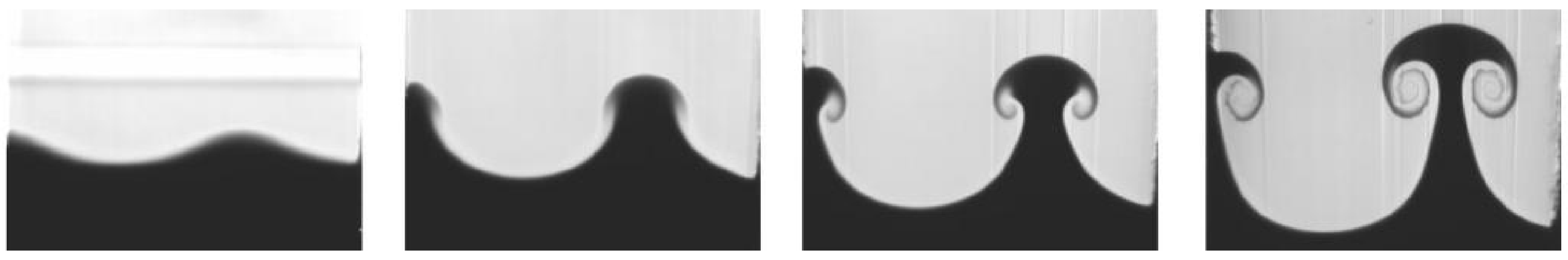
first light

Motivation

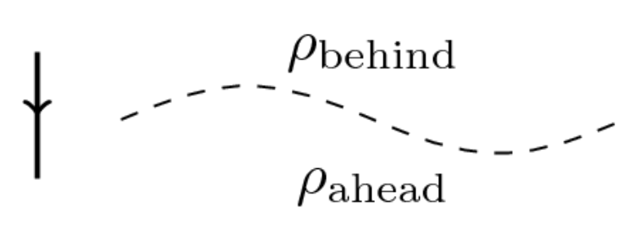
- First Light Fusion (FLF) is investigating projectile-driven inertial confinement fusion.
- Projectiles are driven to velocities of 10-20 km/s by electromagnetic launch on our pulsed power machine M3.
- Hyper-velocity projectiles drive strong shocks across material interfaces in our targets.
- Therefore, the Richtmyer-Meshkov instability (RMI) may be an important factor in determining target performance.
- Modelling is performed using two in-house codes: Hytrac (2D AMR w/ front-tracking) and B (3D multimaterial resistive-MHD)
- RMI benchmarking is important for:
  1) validation of hydrodynamic models
  2) assessing RMI impact on target designs
- **Our aim is to develop an RMI modelling capability valid over a range of target setups and suitable for reproducible CI testing.**



t = 0.26 ms   t = 1.16 ms   t = 2.26 ms   t = 3.46 ms

Fig. 1: (above) Experimental data (PLIF images) from the shocktube experiments of Jacobs & Krivets [3] illustrating the evolution of a single-mode Richtmyer-Meshkov instability; the images above show an Air-SF6 interface for positive Atwood no. at M_i = 1.3 and λ = 5.9cm. The instability typically evolves in four phases: (1) linear growth, (2) bubble/spike formation, (3) K-H instability seeding, and (4) fine-scale instability formation.

- The R-M instability [1] arises when fluid interfaces are impulsively accelerated, i.e., when shocks traverse material interfaces.



$$A = \frac{\rho_{\text{ahead}} - \rho_{\text{behind}}}{\rho_{\text{ahead}} + \rho_{\text{behind}}}$$

- Instability character is defined by the Atwood number; Unlike the Rayleigh-Taylor instability (RTI), the RMI grows for both +ve and -ve A.
- We are benchmarking against experimental Air-SF_6 shocktube [2,3] and Be-AGAR NOVA laser results [4], due to the well-documented conditions and high quality data available.
- The Air-SF_6 test has A = 0.62 and is shocked at M_i = 1.3. The Be-AGAR test has A = -0.87 and M_i ≈ 9-18. These conditions explore a diverse range of shock phenomena over different time/space scales.

Implementation of analytical RMI models

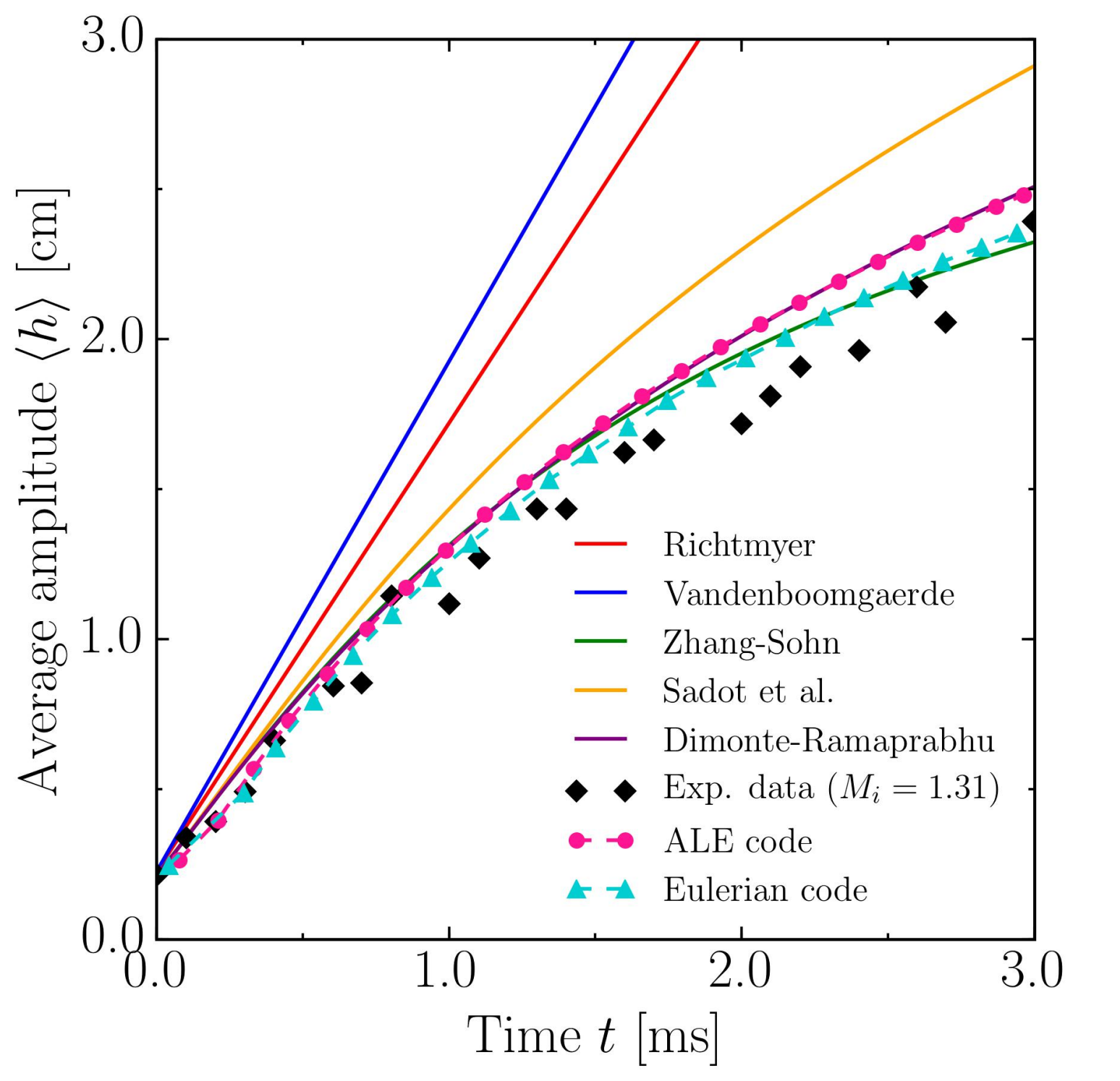


Fig. 2: Model comparison vs the Tri-Lab test suite RMI test, i.e., positive Atwood no. test. Data points digitised from [5]. The D-R and Zhang-Sohn models provide the best match to simulations and experimental data.

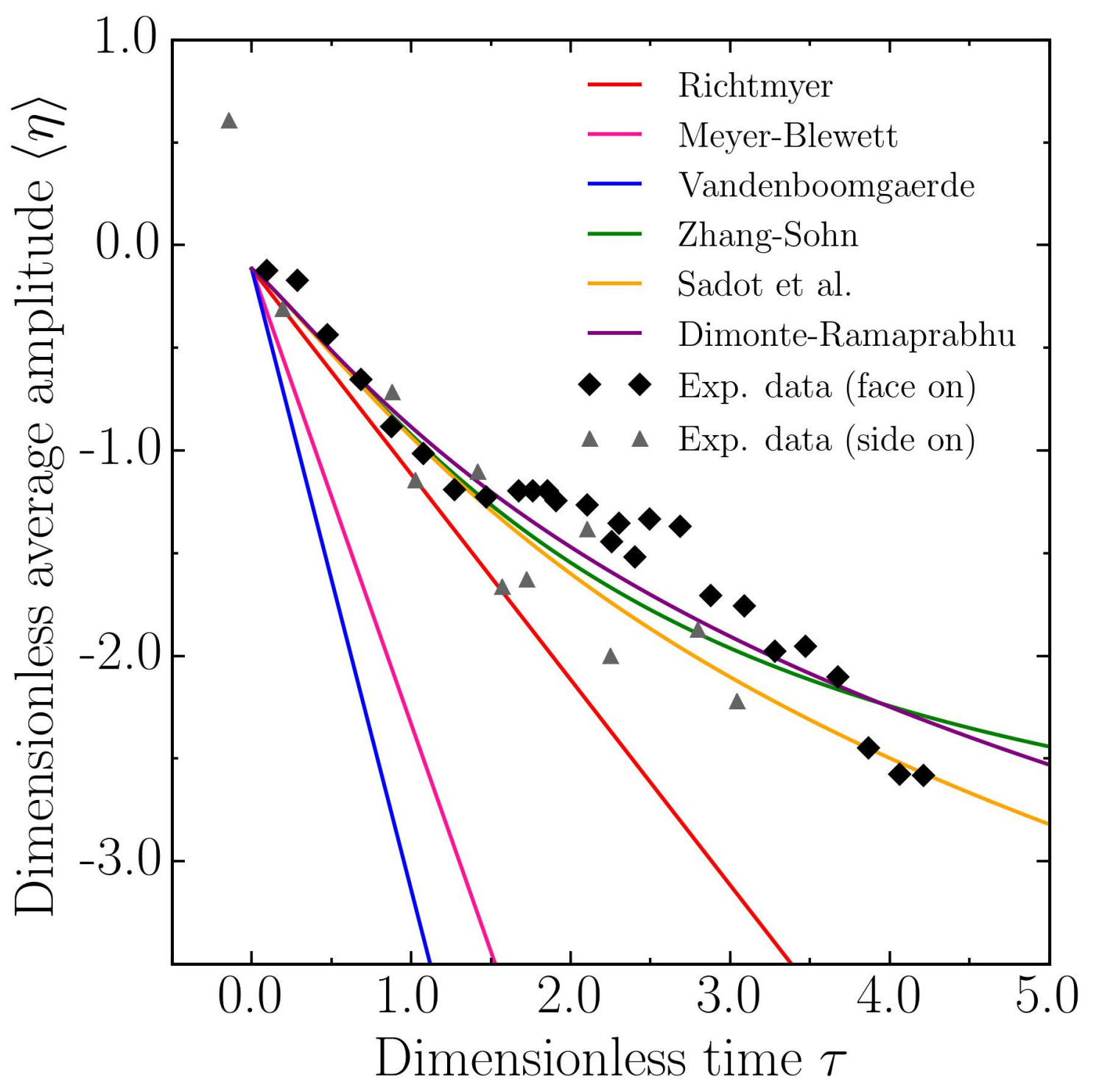


Fig. 3: Model comparinson vs the Dimonte-Remington (fast) case, i.e., negative Atwood no. In this case the three non-linear models are all comparable to the experiment result, but the data spread is too large to clearly identify the best.

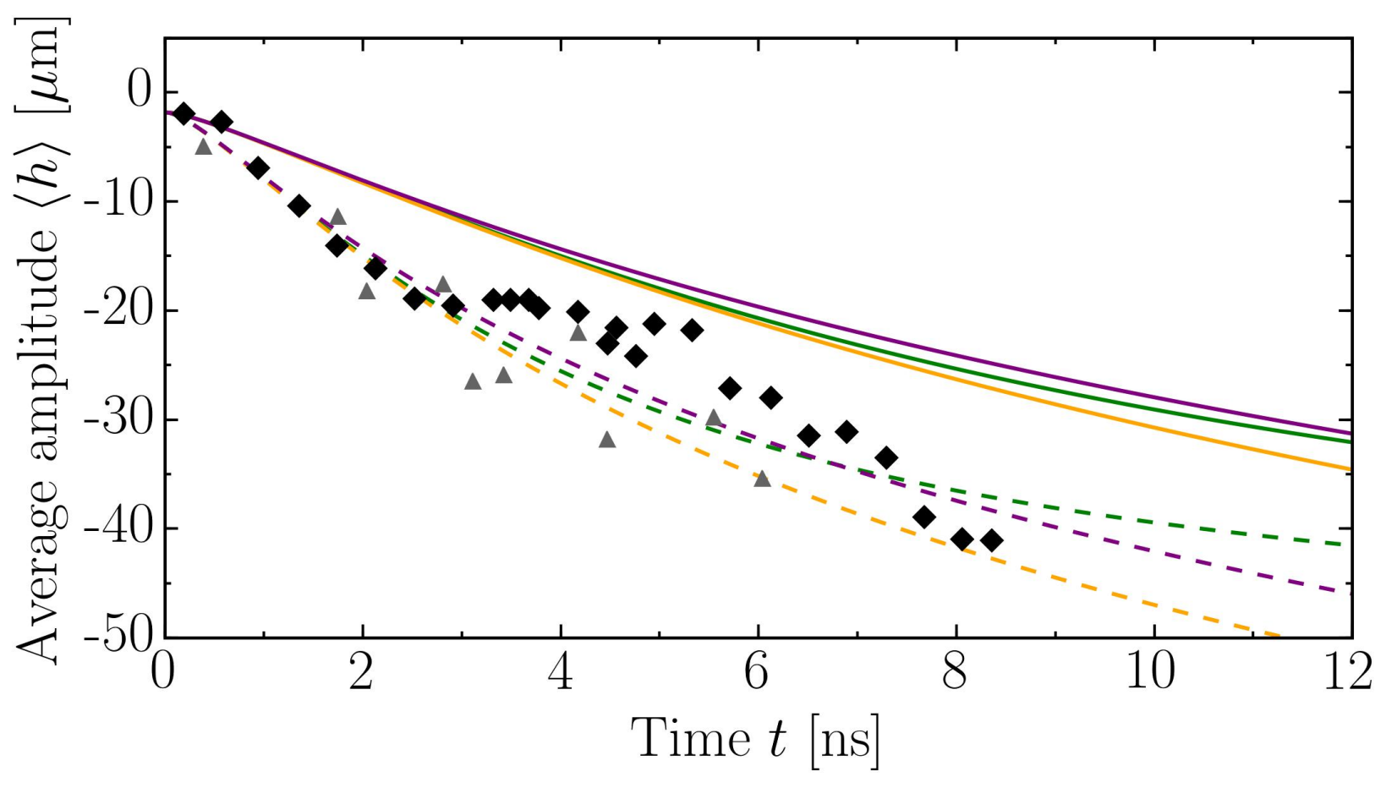


Fig. 4: A comparison of the non-linear models against the D-R NOVA data, as initialised w/ the Richtmyer (solid) and the Meyer-Blewett (dashed) models. The Meyer-Blewett-initialised models provide a better match to the data.

- Analytical models comprise three catergories; impulsive, linear and non-linear (see Table 1).
- The simplest analytical model is the impulsive Richtmyer [1] setup - found from the impulsive limit of Taylor's linear RTI growth equation:
- The Tri-Lab test suite (TLTS) [5] RMI verifcation test has been performed (Fig. 2). This consists of a perturbed Air-SF_6 interface under conditions of [2], but with P = 20 MPa.
- Fig. 3 compares models under experimental Dimonte- Remington [4] NOVA conditions.
- No significant difference was observed using Be Ideal gas EoS and FEOS.
- Fig. 4 shows the importance of the linear growth rate used to initialise the non-linear model.

Model		Validity / Notes
Richtmyer (R) [1]	Impulsive	For reflected shocks (RS) - use post-shock values: $V_R(t) = k u_c h_0^+ A^+$.
Meyer-Blewett (MB) [6]	Impulsive	Valid for RS **and** reflected rarefactions: $V_{\text{MB}}(t) = \frac{1}{2} k u_c \left(h_0^+ + h_0^-\right) A^+$.
Dimonte-Ramaprabhu (DR) [11]	Non-linear	Non-linear model reproduces asymptotic growth rate at early and late times.

Table 1: A summary of different RMI models considered in this benchmarking work and their notable features.

- The Dimonte-Ramaprabhu growth rate is:

$$V_{\text{b/s}} = V_0 \frac{1 + \left[2 - F_{\text{b/s}}\right]\tau}{1 + C_{\text{b/s}}\tau + \left[2 - F_{\text{b/s}}\right] F_{\text{b/s}}\tau^2} \begin{cases} F_{\text{b/s}}\left(|A^+|\right) = 1 \pm |A^+| \\ C_{\text{b/s}}\left(|A^+|, |\eta_0^+|\right) = \frac{1}{4}\left[\frac{7}{2} + F_{\text{b/s}}\left(|A^+|\right) - \left(3 - F_{\text{b/s}}\left(|A^+|\right)\right)|\eta_0^+|\right] \end{cases}$$

- Integrating yields the D-R model amplitude change:

$$\Delta\eta_{\text{b/s}}(t) = \text{sgn}(V_0)\left[a_{\text{b/s}} J\left(\tau; b_{\text{b/s}}, c_{\text{b/s}}\right) + b_{\text{b/s}} K\left(\tau; b_{\text{b/s}}, c_{\text{b/s}}\right)\right] \begin{cases} J(x;\alpha,\beta) = \dfrac{2}{\sqrt{4\beta - \alpha^2}} \arctan\left(\dfrac{x\sqrt{4\beta-\alpha^2}}{2+\alpha x}\right) \\ K(x;\alpha,\beta) = \ln\left(1 + \alpha x + \beta x^2\right) - \dfrac{\alpha}{2\beta} J(x;\alpha,\beta) \end{cases}$$

RMI simulation configuration

- The simulations presented here were performed with the 2D Hytrac code.
- Planar geometry w/ AMR and front-tracking is used.
- Ideal gas EoS was used in all cases.
- Simulations were initalised using three regions:
  - Two materials in pressure equilibrium separated by a perturbed interface
  - A shocked region defined by the analytical solution to a 1D Riemann problem resulting in the required interface contact velocity u_c
- Resolution (via AMR level), higher order numerical schemes and front-tracking methods were varied.
- The average perturbation amplitude was obtained by post-processing the bubble and spike features.
- Moving-window simulations, i.e., in the reference frame of the contact, were attempted but were found to yield unstable interfaces at low velocities (~100 m/s), as previously noted in [12].
- The simulation parameters are presented in Table 2.

Parameter	Tri-Lab test suite (Air-SF_6)	Dimonte-Remington (Be-AGAR)	
		Slow	Fast
γ_A	1.276	1.80	
γ_B	1.093	1.45	
ρ_A	1.3 [kg/m³]	1700 [kg/m³]	
ρ_B	5.5 [kg/m³]	120 [kg/m³]	
A^-	0.616	-0.868	
P	909 [bar]	0.1 [MBar]	
ϵ	0.441	0.991	0.998
M_i	1.3	9.1	18.2
λ	5.93 [cm]	100 µm	
h_0^-	3.00 [mm]	10 µm	14 µm
u_c	97.0 [m/s]	35 [km/s]	71 [km/s]

Table. 2: The parameters used to setup the Tri-Lab test suite and Dimonte-Remington simulations.

Numerical results and analysis

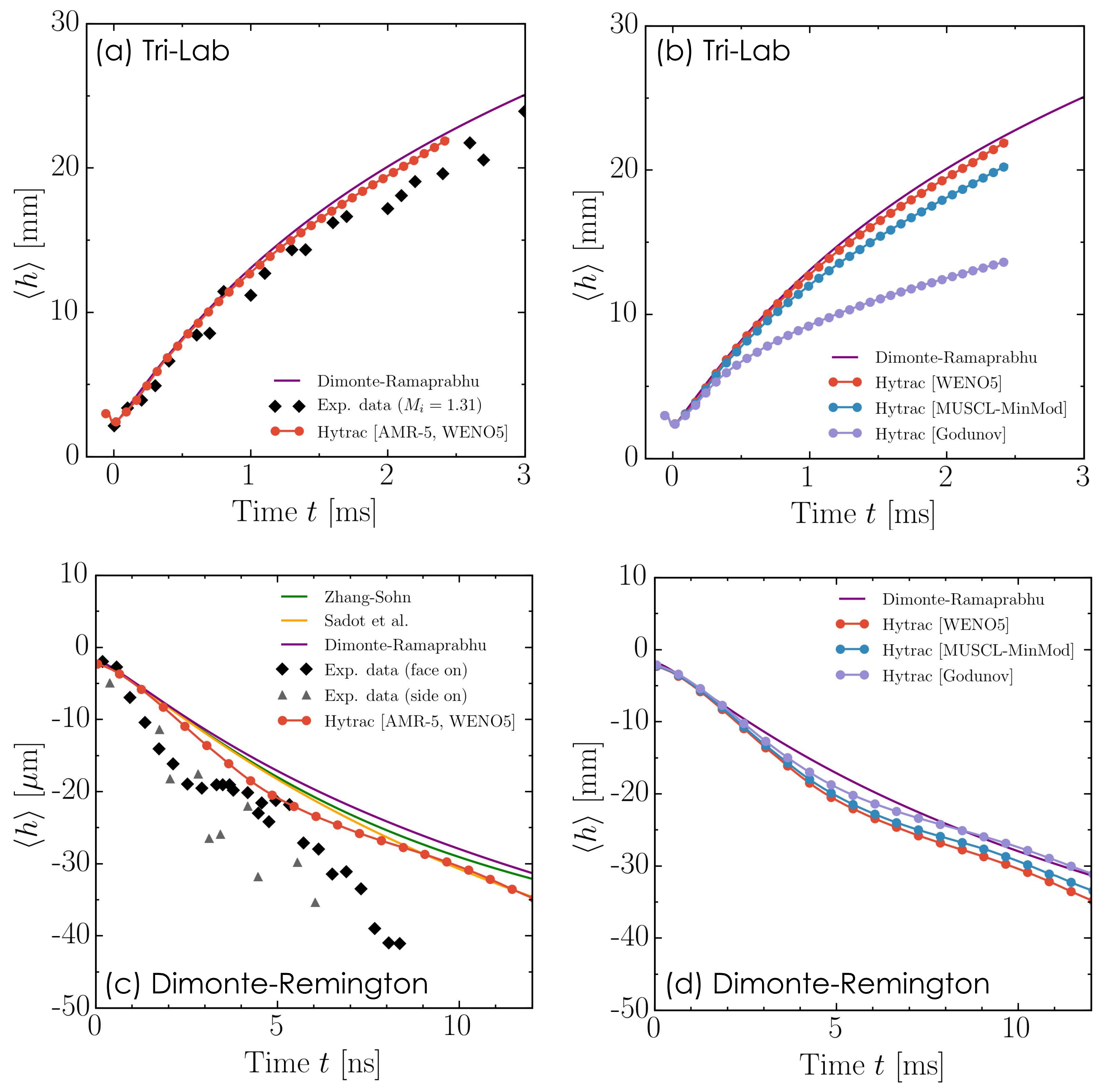


Fig. 5: (right) A comparison of the instability amplitudes for the TLTS and D-R cases as simulated using Hytrac, compared to the analytical models and exp. data. **(a)** The average amplitude from Hytrac using the WENO5 scheme compares favourably to the Dimonte-Ramaprabhu model and the shocktube data. **(b)** Higher order numerical schemes - Godunov (1st order), MUSCL (2nd order), WENO5 (5th order) - converge to the model more rapidly at fixed resolution. **(c)** For the D-R test, the Hytrac average amplitudes are in line with the models, but there is still some discrepancy from the exp. data. **(d)** Increasing order numerical schemes are compared for the D-R (fast) test. The variation is found to be smaller than for the TLTS case.

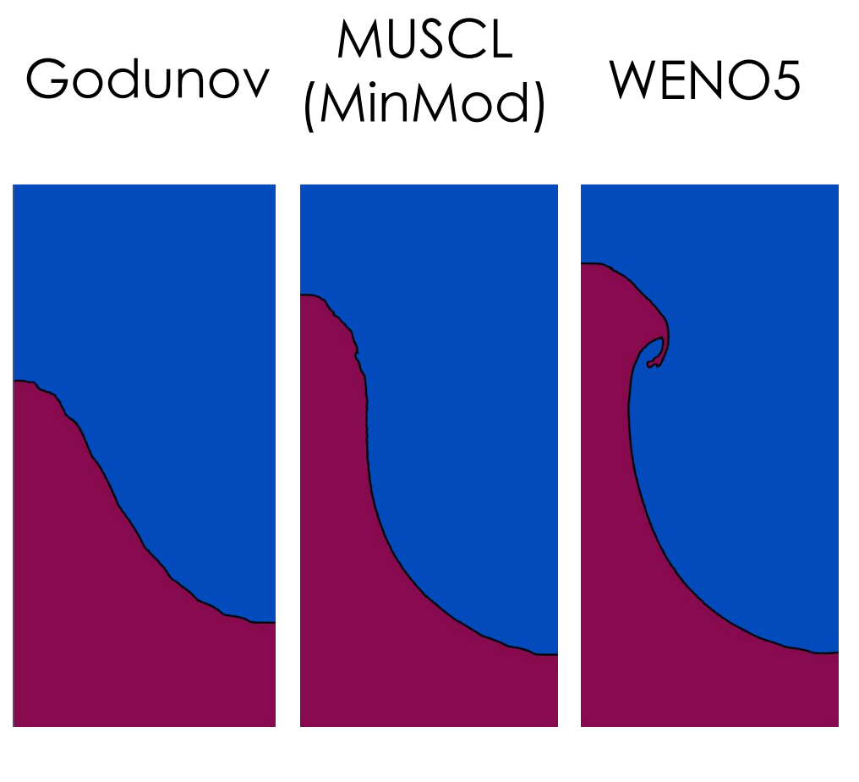


Fig. 7: (above) Interface comparison for the TLTS case after t = 2.5 ms of simulation at n=10, AMR-5 (=> dx=180 µm. i.e., 320 cells per λ. The 'roll-up' feature appears for higher order numerical schemes.

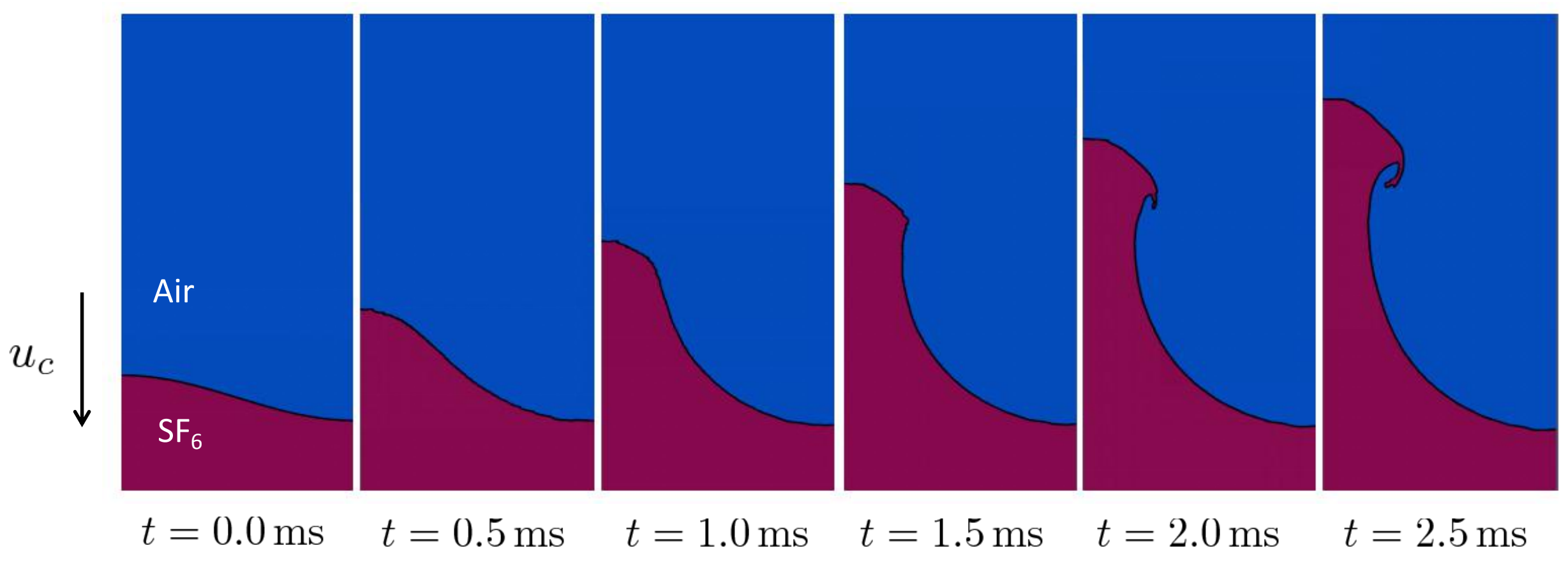


Fig. 6: (above) Time evolution of the instability for the Tri-Lab case. As shown in the above figure, these images correspond to Hytrac run with AMR-5, WENO5.

- The Hytrac RMI comparison for the TTS case is good at higher AMR levels using higher order numerical schemes. Both the Dimonte-Ramaprabhu model and experiemetal data are well matched.
- The effect of the front-tracking scheme on interface evolution is of continued interest.
- For the more extreme Dimonte-Remington conditions, Hytrac matches the analytical models initialised w/ the Richtmyer growth rate reasonably well, but there is a discrepancy with the experimental data.
- Matching the initial conditions for the radiative precursor is a challenging problem.

Conclusions and future work

- We have performed Richtmyer-Meshkov instability simulations to validate Hytrac.
- The TLTS and D-R tests have been incorporated into the Hytrac CI suite.
- A good match to existing literature for both positive (Air-SF_6) and negative (Be-AGAR) tests is found.
- This establishes trust in our modelling tools to evaluate novel, target-relevant parameters.
- **Future steps: 1) validate the B code, 2) multi-mode RMI tests, 3) perform integrated target simulations, and 4) consider in-house experiments to provide new code validation data.**

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